



## Using the original and ‘symmetrical face’ training samples to perform representation based two-step face recognition

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### ABSTRACT

A limited number of available training samples have become one bottleneck of face recognition. In real-world applications, the face image might have various changes owing to varying illumination, facial expression and poses. However, non-sufficient training samples cannot comprehensively convey these possible changes, so it is hard to improve the accuracy of face recognition. In this paper, we propose to exploit the symmetry of the face to generate new samples and devise a representation based method to perform face recognition. The new training samples really reflect some possible appearance of the face. The devised representation based method simultaneously uses the original and new training samples to perform a two-step classification, which ultimately uses a small number of classes that are ‘near’ to the test sample to represent and classify it and has a similar advantage as the sparse representation method. This method also takes advantages of the score level fusion, which has proven to be very competent and usually performs better than the decision level and feature level fusion. The experimental results show that the proposed method outperforms state-of-the-art face recognition methods including the sparse representation classification (SRC), linear regression classification (LRC), collaborative representation (CR) and two-phase test sample sparse representation (TPTSSR).

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### 1. Introduction

As one of the most active branches of biometrics, face recognition is attracting more and more attention [1–5]. In the past, various face recognition algorithms have been devised [6–9]. However, up to now, face recognition is still faced with a number of challenges such as varying illumination, facial expression and poses [10–16]. It seems that more training samples are able to reveal more possible variation of the illumination, facial expression and poses and are consequently beneficial for correct classification of the face. However, in real-world applications, there are usually only a limited number of available training samples. This is mainly because a face recognition system usually has limited storage space and captures training samples in a short time. In some special cases such as the personal identity card based face recognition, there is even only one training sample per

subject. Non-sufficient training samples indeed have become one bottleneck of face recognition [17–20].

In order to obtain better face recognition result, the literatures have proposed to synthesize new samples from the true face images. For example, Tang et al. [21] used prototype faces and an optic flow and expression ratio image based method to generate ‘virtual’ facial expression. Thian et al. [22] used simple geometric transformations to generate virtual samples. Ryu et al. [23] exploited the distribution of the given training set to generate virtual training samples. Beymer et al. [24] and Vetter et al. [25] synthesized new face samples with virtual views. Jung et al. [26] exploited the noise to synthesize new face samples. Sharma et al. [27] synthesized multiple virtual views of a person under different poses and illumination from a single face image and exploited extended training samples to classify the face. In order to overcome the small sample size problem of face recognition, Liu et al. [28] represented each single image as a subspace spanned by its synthesized (shifted) samples. From the viewpoint of applications, the ways to generate virtual face images can be categorized into two kinds, i.e. the way to generate two-dimensional virtual face images and the way to construct three-dimensional virtual face images [29].

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The one training sample issue has also emerged as an active research sub-area of face recognition in recent years, and many ad hoc techniques have been proposed for this problem [30]. For example, previous literatures have used probabilistic matching [31–33] and neural network methods [34–36] for the one training sample issue. Qiao et al. [37] and Liu et al. [28] proposed sparsity preserving discriminant analysis and the single image subspace method for the one training sample issue respectively. Moreover, synthesizing virtual samples has also been used as a means to cope with the one training sample issue [23,24,38,39].

We note that the face has a symmetrical structure. Not only the facial structure but also the facial expression is symmetry [39]. The symmetry property has been successfully applied to face detection [40–42]. In face detection, the symmetry property of the human face is very useful to quickly locate the candidate faces [41].

In this paper, we propose to exploit the symmetry of the face to generate new training samples and devise a representation based method to perform face recognition. The new training samples indeed reflect some possible appearance of the face. The devised representation based method simultaneously uses the original and new training samples to perform a two-step classification. This method also takes advantages of the score level fusion, which has proven to be very competent and is usually better than the decision level and feature level fusion.

The two-step face recognition (TSFR) algorithm proposed in this paper is indeed a representation based classification (RBC) algorithm. A number of works have shown that the RBC algorithm can obtain a very high accuracy for image classification such as face recognition [43–49]. However, the conventional RBC algorithm such as the one proposed in [43,44] has a relatively high computational cost. Our TSFR algorithm is mathematically tractable and computationally efficient. The first step of our algorithm aims at identifying and discarding the classes whose training samples are ‘far’ from the test sample and the second step then exploits the training samples of the remaining classes to determine the class label of the test sample. The experimental results show that the proposed method can classify the face with a high accuracy and outperforms the state-of-the-art face recognition methods.

## 2. The proposed method

In this section we present the main steps of the proposed method in detail. Suppose that there are  $c$  classes and each class has  $n$  training samples. Let  $x_1, \dots, x_N$  represent all the  $N$  training samples ( $N=nc$ ).

### 2.1. Main steps of the proposed method

The proposed method includes the following main steps. The first step generates ‘symmetrical face’ training samples. The second and third steps use the original and ‘symmetrical face’ training samples to perform two-step face recognition, respectively. The algorithm of two-step face recognition will be described in Section 2.2. The fourth step combines the scores obtained using the second and third steps to conduct weighted score level fusion, getting the ultimate classification result. We present these steps as follows:

Step 1. Use every original training sample to generate two ‘symmetrical face’ training samples. Let  $x_i \in \mathbb{R}^{p \times q}$  be the  $i$ -th training sample in the form of image matrix. Let  $y_i^1$  and  $y_i^2$  respectively stand for the first and second ‘symmetrical face’ training samples generated from  $x_i$ . The left half columns of  $y_i^1$

is set to the same as that of  $x_i$  and the right half columns of  $y_i^1$  is the mirror image of the left half columns of  $y_i^1$ . However, the right half columns of  $y_i^2$  is set to the same as that of  $x_i$  and the left half columns of  $y_i^2$  is the mirror image of the right half columns of  $y_i^2$ . The mirror image  $S$  of an arbitrary image  $R$  is defined as  $S(i,j)=R(i,V-j+1), i=1, \dots, U, j=1, \dots, V$ .  $U$  and  $V$  stand for the numbers of the rows and columns of  $R$ , respectively.  $S(i,j)$  denotes the pixel located in the  $i$ th row and  $j$ th column of  $S$ .

Step 2. Use the original training samples to perform two-step face recognition. Let  $s_j^1$  denote the score of test sample  $z$  with respect to the  $j$ -th class. For the algorithm, please see Section 2.2.

Step 3. Use the ‘symmetrical face’ training samples to perform two-step face recognition. Let  $s_j^2$  denote the score of test sample  $z$  with respect to the  $j$ -th class. This step shares the same algorithm as Step 2.

Step 4. Combine the scores obtained using the second and third steps to conduct weighted score level fusion. For test sample  $z$ , we use  $s_j = w_1 s_j^1 + w_2 s_j^2$  to calculate the ultimate score with respect to the  $j$ -th class.  $w_1$  and  $w_2$  are the weights. Let  $w_1 + w_2 = 1$  and  $w_2$  be smaller than  $w_1$ .

### 2.2. The algorithm of two-step face recognition

In this subsection we present the algorithm of two-step face recognition in detail. This algorithm first coarsely determines a small number of candidate classes of the test sample and then finely identifies the class that the test sample is the most similar to.

Before this algorithm is employed, all the samples should be converted into one-dimensional column vectors. We use  $\tilde{x}_i, \tilde{y}_i^1, \tilde{y}_i^2, \tilde{z}$  to denote the one-dimensional column vectors of  $x_i, y_i^1, y_i^2, z$  respectively.

For simplicity of presentation, we describe only the algorithm on the original training samples. The algorithm on the ‘symmetrical face’ training samples is the same except that the original training samples are replaced with the ‘symmetrical face’ training samples. The algorithm first assumes that the following equation is approximately satisfied:

$$\tilde{z} = a_1 \tilde{x}_1 + \dots + a_N \tilde{x}_N, \quad (1)$$

$a_i$  is referred to as the coefficient. Let  $X = [\tilde{x}_1 \dots \tilde{x}_N]$  and  $A = [a_1 \dots a_N]^T$ . We rewrite Eq. (1) as

$$\tilde{z} = XA, \quad (2)$$

$A$  is calculated using  $\bar{A} = (X^T X + \mu I)^{-1} X^T \tilde{z}$ .  $\bar{A} = [\bar{a}_1, \dots, \bar{a}_N]^T$ .  $\mu$  is a small positive constant and  $I$  is the identity matrix.

Eq.(1) implies that the effect on representing the test sample of the  $k$ -th class can be evaluated using

$$d_k = \|\tilde{z} - \sum_{i=(k-1)n+1}^{kn} \bar{a}_i \tilde{x}_i\|. \quad (3)$$

It is clear that  $\tilde{x}_{(k-1)n+1}, \dots, \tilde{x}_{kn}$  are training samples of the  $k$ -th class and  $\bar{a}_{(k-1)n+1}, \dots, \bar{a}_{kn}$  are the corresponding coefficients. We would like to point out that the effect on representing the test sample of the  $k$ -th class is somewhat similar to the distance between the test sample and the  $k$ -th class. As a result, the effect on representing the test sample of the  $k$ -th class can be also evaluated by others means such as the sum of the Euclidean distances between the test sample and each training sample from the  $k$ -th class.

If  $d_{r_1} \leq d_{r_2} \leq \dots \leq d_{r_t}$ , then we say that the  $r_1$ -th,  $r_2$ -th, ...,  $r_t$ -th classes are the first  $t$  candidate classes of the test sample. In other words, we can consider that the ultimate class label of the test sample should be one element of  $D = c_{r_1}, c_{r_2}, \dots, c_{r_t}$ .

$c_{r_1}, c_{r_2}, \dots, c_{r_t}$  are the class labels of the first  $t$  candidate classes, respectively. As the above steps roughly determine that the test sample is from a small number of classes, we refer to them as coarse classification.

The algorithm then uses a linear combination of the training samples of the first  $t$  candidate classes to represent the test sample. In other words, if the training samples of the first  $t$  candidate classes are denoted by  $\tilde{x}'_1, \dots, \tilde{x}'_{tm}$ , respectively, then the algorithm assumes that the following equation is approximately satisfied:

$$\tilde{z} = f_1 \tilde{x}'_1 + \dots + f_{tm} \tilde{x}'_{tm}, \tag{4}$$

where  $f_i$  is the coefficient. We rewrite Eq.(4) as

$$\tilde{z} = X'F, \tag{5}$$

where  $F = [f_1 \dots f_{tm}]^T$ ,  $X' = [\tilde{x}'_1, \dots, \tilde{x}'_{tm}]$ .  $F$  is calculated using

$$\bar{F} = (X'^T X' + \gamma I)^{-1} X'^T \tilde{z} \tag{6}$$

$\bar{F} = [\bar{f}_1 \dots \bar{f}_{tm}]^T$ .  $\gamma$  is a small positive constant and  $I$  also denotes the identity matrix.

Suppose that  $\tilde{x}'_g, \dots, \tilde{x}'_h$  stand for all the training samples of the  $r$ -th class ( $r \in c_{r_1}, c_{r_2}, \dots, c_{r_t}$ ) and the coefficients are  $\bar{f}_g, \dots, \bar{f}_h$ , respectively. The ultimate effect on representing the test sample of the  $r$ -th class can be evaluated using

$$u_r = \|\tilde{z} - \sum_{i=g}^h \bar{f}_i \tilde{x}'_i\|. \tag{7}$$

If  $k = \operatorname{argmin} u_r$ , then test sample  $\tilde{z}$  is ultimately assigned to the  $k$ -th class, which is also referred to as the result of fine classification. The code of the proposed method can be downloaded at (<http://www.yongxu.org/lunwen.html>).

### 3. Analysis of the proposed method

In this section we show the rationales of the proposed method. First, the 'symmetrical face' training samples in the proposed method indeed reflect some possible appearance of the face, which are not shown by the original training samples. Fig. 1 shows some original training samples from the ORL face database and the 'symmetrical face' training samples generated from the original training samples. Fig. 2 shows some original training samples from the FERET face database and the corresponding

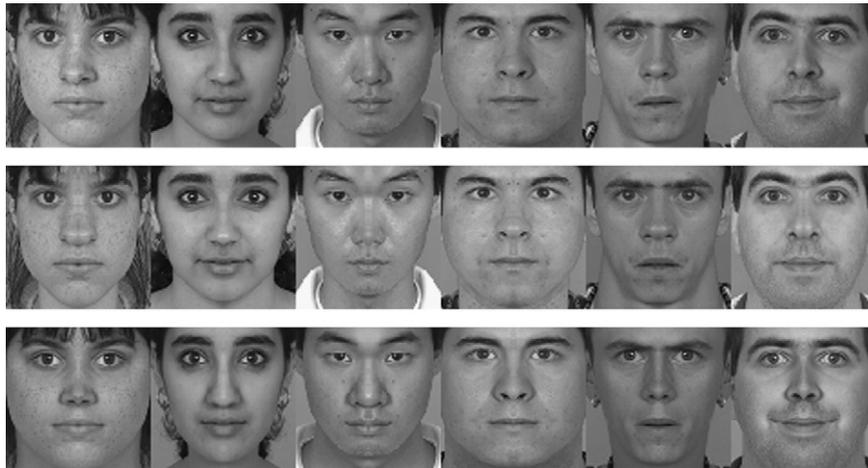
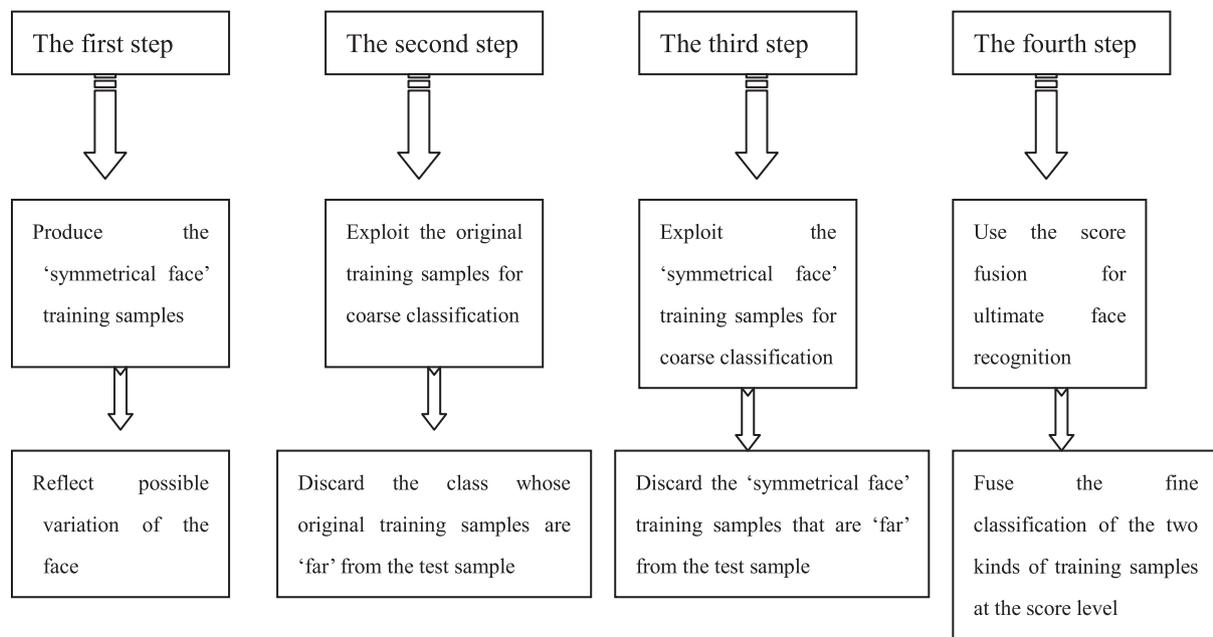
'symmetrical face' training samples. Fig. 3 shows some original training samples from the AR face database and the corresponding 'symmetrical face' training samples. We see that the 'symmetrical face' training samples not only seem to be different from the original training samples, but also indeed somewhat reflect the possible variation of the face in image scale, pose and illumination. Thus, 'symmetrical face' training samples are very useful to overcome the issue of non-sufficient training samples. In the real world, the image scale is variable owing to the various distances between the person and camera. Fig. 4 shows a test sample (4(a)) that is erroneously and correctly classified by the collaborative representation (CR) method proposed in [47] and our method, respectively. As the collaborative representation method exploits only the original training samples and our method uses both the original training samples and 'symmetrical face' training samples, the 'symmetrical face' training sample is really beneficial for correct classification of the test sample. During the register phase in real-world applications, after the face image is obtained by the face detection procedure, the 'symmetrical face' training samples can be easily and efficiently generated. Since the 'symmetrical face' training samples are complementary for the original training samples, the system can capture only a few original training samples and can still obtain enough information of the face. To capture only a few original training samples will also allow the system to take only a short time to complete the register phase.

The second rationale of the proposed method is that it respectively uses the 'symmetrical face' training samples and the original training samples to obtain the scores of the test sample with respect to different classes and properly exploits a weighted fusion scheme to combine them for ultimate face recognition. As the 'symmetrical face' training samples contain less information than the original training samples, it is very reasonable for the proposed method to assign a smaller weight to the 'symmetrical face' training samples.

The third rationale of the proposed method is that it uses the two-step face recognition, which is able to reduce the side-effect on classification, of the test sample, of the classes that are very dissimilar to the test sample. Actually, the literature has shown that the test sample is usually not from these classes [45]. As a result, by eliminating these classes the fine recognition can increase the probability of the test sample being correctly classified and can achieve a higher accuracy. The significance of every step of the proposed method can be briefly described below.



Fig. 1. Some original training samples from the ORL face database and the corresponding 'symmetrical face' training samples. The first row shows the original training samples. The second and third rows respectively show the first and second 'symmetrical face' training samples generated from the original training sample.



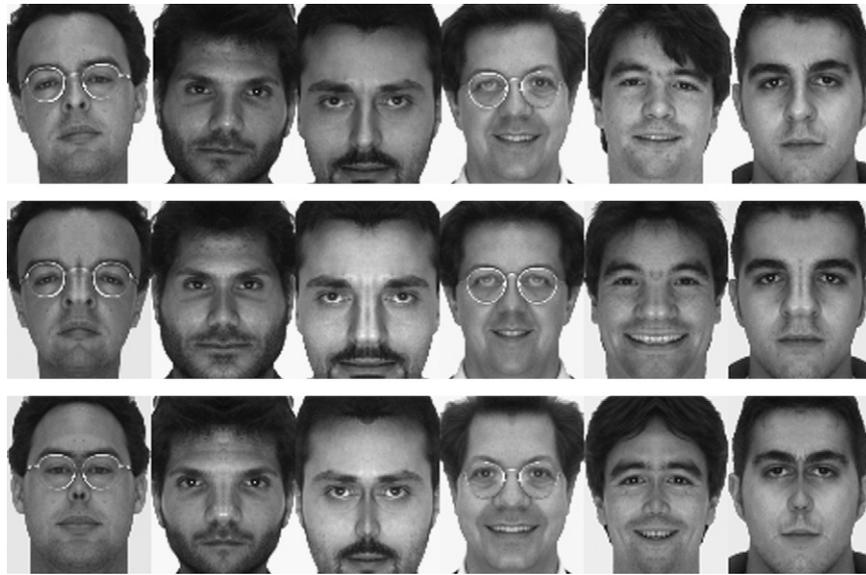
**Fig. 2.** Some original training samples from the FERET face database and the corresponding 'symmetrical face' training samples. The first row shows the original training samples. The second and third rows respectively show the first and second 'symmetrical face' training samples generated from the original training sample.

In order to assess the capability, to represent the test sample, of our method, we define representation error as follows. Let  $z$  be the test sample. For a RBC method, let  $P$  and  $s$  be the matrix consisting of all the available training samples and the solution vector of the corresponding RBC, respectively.  $d_z = \|z - Ps\|$  is referred to as the representation error of  $z$ .  $Ps$  is referred to as representation result of  $z$ . It is clear that the smaller the  $d_z$  is the more the representation result approximates the test sample. The analysis also shows that in our method, a lower representation error usually does not mean a lower rate of classification errors. This is true for both the RBC on the original training samples and the RBC on the 'symmetrical face' training samples. Figs. 5 and 6 show the representation errors of the test sample obtained using the coarse classification and fine classification based on the original training samples and 'symmetrical face' training samples, respectively. These figures are obtained under the condition that the first 14 face images of each subject in the AR face database are used as original training samples and the remaining face images are taken as test samples. We see that the coarse classification based on the original training samples and 'symmetrical face' training samples always leads to a lower representation error

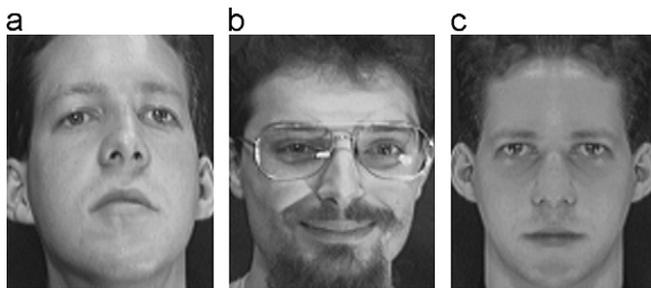
than the corresponding fine classification. However, these two figures show that the fine classification produces fewer classification errors than the coarse classification. Actually, as shown in the experimental section, the fine classification always obtains a lower rate of classification errors. This tells us that for a RBC, the key is not to produce a low representation error but to make the class that the test sample is truly from the most similar with the test sample, having the minimum deviation from the test sample among all the classes. Though the fine classification is not optimal for providing a good representation for the test sample, it performs very well in making the test sample more similar with the test sample. As shown early, it achieves this by discarding the classes that are 'far' from the test sample. Usually, there is also a large probability of the test sample being not truly from these classes.

#### 4. Experimental results

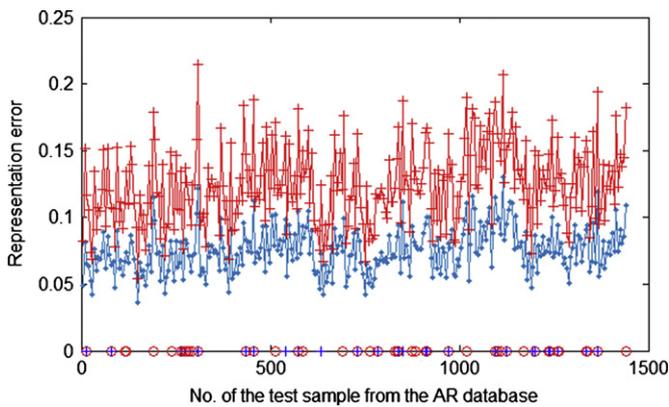
We used the FERET, AR and ORL face databases to conduct experiments. For the FERET and ORL databases, we show the experimental results under the conditions that the weights of the



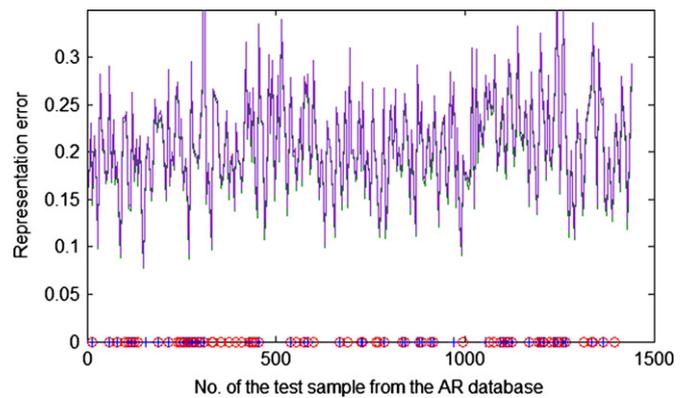
**Fig. 3.** Some original training samples from the AR face database and the corresponding 'symmetrical face' training samples. The first row shows the original training samples. The second and third rows respectively show the first and second 'symmetrical face' training samples generated from the original training sample.



**Fig. 4.** A test sample that is erroneously and correctly classified by the collaborative representation and our method, respectively. (a) Test sample. (b) Original training sample which is the nearest to the test sample. (c) 'Symmetrical face' training sample which is the nearest to the test sample among all the 'symmetrical face' training samples.



**Fig. 5.** Representation errors of the test sample obtained using the coarse classification and fine classification based on the original training samples. The blue and red lines respectively depict the representation errors obtained using the coarse classification and fine classification. In the horizontal axis, the red circle and blue '+' respectively means that the corresponding test sample is erroneously classified by the coarse classification and fine classification based on the original training samples. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)



**Fig. 6.** Representation errors of the test sample obtained using the coarse classification and fine classification based on the 'symmetrical face' training samples. The green and purple lines respectively depict the representation errors obtained using the coarse classification and fine classification. In the horizontal axis, the red circle and blue '+' respectively means that the corresponding test sample is erroneously classified by the coarse classification and fine classification based on the 'symmetrical face' training samples. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

**Table 1**  
Experimental results on the AR database.

Number of the original training samples per class	13	14	15	16
The proposed method ( $w_1=0.75$ ) (%)	23.72	9.93	8.03	6.83
CR on the original training samples (%)	28.97	14.86	11.82	9.83
CR on the 'symmetrical face' training samples (%)	36.15	27.15	22.27	18.83
Coarse score fusion (%)	28.33	14.44	11.74	9.58
SRC (%)	32.37	16.74	16.82	17.42
TPTSSR (%)	24.81	9.72	8.33	7.17
LRC (%)	34.36	13.13	12.58	13.25
Feature space-based representation method ( $\sigma=1.0e6$ ) [49] (%)	39.36	25.07	24.47	25.42
Combination of two-step classification and the feature space-based representation method ( $\sigma=1.0e6$ ) (%)	39.23	25.07	24.47	25.33

score fusion were set to  $w_1=0.85$  and  $w_2=0.15$ ,  $w_1=0.75$  and  $w_2=0.25$ ,  $w_1=0.65$  and  $w_2=0.35$ , respectively. For the AR database, because the experiment is very time consuming, we conduct only the experiment on  $w_1=0.75$  and  $w_2=0.25$ . Both  $\mu$  and  $\gamma$  were set to 0.01. Each sample was converted into a unit vector with length of 1 in advance. Tables 1, 2 and 3 show the rate of classification errors under the condition that  $0.3^*c$  candidate classes were used. Besides our proposed method was tested, several state-of-the-art face recognition methods such as two-phase test sample sparse representation (TPTSSR) [45], collaborative representation (CR) proposed in [47], the feature space-based representation method proposed in [49], sparse representation classification (SRC) proposed in [50] and linear regression classification (LRC) [51] were also tested. As the performance of the feature space-based representation method is directly related to parameter  $\sigma$ , we set it to different values and show the best classification result and the corresponding value in the table.

**Table 2**  
Experimental results on the ORL database.

Number of the original training samples per class	1	2	3
The proposed method ( $w_1=0.85$ ) (%)	24.72	10.31	10.00
The proposed method ( $w_1=0.75$ ) (%)	24.17	10.31	8.93
The proposed method ( $w_1=0.65$ ) (%)	24.72	10.63	9.29
CR on the original training samples (%)	31.39	16.56	15.00
CR on the 'symmetrical face' training samples (%)	34.72	23.13	20.71
Coarse score fusion (%)	30.28	16.25	13.57
SRC (%)	26.67	15.00	14.29
TPTSSR (%)	26.39	13.44	11.43
LRC (%)	32.50	20.94	18.21
Feature space-based representation method( $\sigma=1.0e7$ ) [49] (%)	27.22	11.87	10.71
Combination of two-step classification and the feature space-based representation method ( $\sigma=1.0e7$ ) (%)	27.22	11.25	10.71

**Table 3**  
Experimental results on the FERET database.

Number of the original training samples per class	1	2	3
The proposed method ( $w_1=0.85$ ) (%)	50.42	35.30	41.88
The proposed method ( $w_1=0.75$ ) (%)	48.75	34.80	41.63
The proposed method ( $w_1=0.65$ ) (%)	48.08	32.80	38.25
CR on the original training samples (%)	55.67	41.60	55.63
CR on the 'symmetrical face' training samples (%)	58.33	39.20	41.38
Coarse score fusion (%)	55.67	40.00	52.25
SRC (%)	50.25	35.20	40.00
TPTSSR (%)	52.17	38.70	46.88
LRC (%)	55.08	36.80	42.88
Feature space-based representation method( $\sigma=1.0e6$ ) [49] (%)	56.25	43.40	50.50
Combination of two-step classification and the feature space-based representation method ( $\sigma=1.0e6$ ) (%)	54.25	41.70	48.50

**Table 4**  
Variation of the rate of classification errors of our method ( $w_1=0.75$  and  $w_2=0.25$ ) with the number of candidate classes.

The ORL database (3 training sample per subject)				
Number of candidate classes	10	20	30	40
The rate of classification errors (%)	10.00	9.29	8.93	12.50
The FERET database (3 training sample per subject)				
Number of candidate classes	50	100	150	200
The rate of classification errors (%)	40.75	45.25	47.13	51.25
The AR database (13 training sample per subject)				
Number of candidate classes	30	60	90	120
The rate of classification errors (%)	23.40	24.42	25.64	28.21

In the experiments, we performed 'coarse score fusion' as follows: CR was respectively first implemented for the original and 'symmetrical face' training samples and then the scores generated from the original and 'symmetrical face' training samples were fused for ultimate face recognition by using the same weight fusion scheme as our proposed method. When implementing TPTSSR, we set parameter  $M=N/2$ , where  $N$  is the number of all the original training samples. We also tested the combination of two-step classification and the feature space-based representation method proposed in [49]. Specifically, the feature space-based representation method first respectively uses the original and 'symmetrical face' training samples to perform coarse classification and then respectively uses them to conduct fine classification. The fine classification was also implemented under the condition that  $0.3^*c$  candidate classes were exploited. Finally, the weighted fusion scheme ( $w_1=0.75$  and  $w_2=0.25$ ) in our method was used to combine the scores generated from the fine classification on the original and 'symmetrical face' training samples and the combined score was used to ultimately classify the test sample.

#### 4.1. Experiments the AR face database

From the AR face database [52], we used 3120 Gray images from 120 subjects, each providing 26 images. These images were taken in two sessions. Every image was resized to a  $50 \times 40$  image. We respectively took the first 13, 14, 15 and 16 face images of each subject as the original training samples and treated the remaining face images as the test samples. The experimental results were shown in Table 1. We see that our method obtains a lower rate of classification errors than all the other methods. For example, when the first 13 face images of each subject were used as the original training samples and the remaining face images were taken as the test samples, the rates of classification errors of our method, CR on the original training samples, SRC, TPTSSR, LRC and the feature space-based representation method are 23.72%, 28.97%, 32.37%, 24.81%, 34.36% and 39.36%, respectively. The fact that coarse score fusion also obtains a higher rate of classification errors than our method also means that to discard the classes that are 'far' from the test sample is really beneficial for the correct classification of the test sample. Moreover, we see that the combination of two-step classification and the feature space-based representation method also performs worse than our method.

#### 4.2. Experiments the ORL face database

The ORL database [53] includes 400 face images taken from 40 subjects, each providing 10 face images. For some subjects, the images were taken at different times, with varying lighting, facial expressions (open/closed eyes, smiling/not smiling), and facial details (glasses/no glasses). Each image was also resized to an image with one half of the original size by using the down-

sampling algorithm. We respectively took the first 1, 2 and 3 face images of each subject as the original training samples and treated the remaining face images as the test samples. The experimental results were shown in Table 2. It shows again that our method performs better than all the other methods. For example, when the first face image of each subject and the remaining face images were respectively used as the original training samples and the test samples, the rates of classification errors of our method ( $w_1=0.75$  and  $w_2=0.25$ ), CR on the original training samples, SRC, TPTSSR, LRC and the feature space-based representation method are 24.17%, 31.39%, 26.67%, 26.39%, 32.50% and 27.22% respectively. We also see that the combination of two-step classification and the feature space-based representation method obtains a higher rate of classification errors than our method.

#### 4.3. Experiments the FERET face database

We used a subset of the FERET face database [54] to test our method. This subset consists of 1400 images from 200 individuals each providing seven images. This subset was composed of images whose names are marked with two-character strings: 'ba', 'bj', 'bk', 'be', 'bf', 'bd', and 'bg'. We resized each image to a  $40 \times 40$  image using the down-sampling algorithm. We respectively took the first 1, 2 and 3 face images of each subject as the original training samples and treated the remaining face images as the test samples. Table 3 shows the rate of classification errors. This table shows that in most cases our method outperforms all the other methods. Table 3 also shows that the combination of two-step classification and the feature space-based representation method obtains a higher rate of classification errors than our method.

#### 4.4. Variation of the performance of our method with the number of candidate classes

In order to comprehensively show the performance of our method, we use Table 4 to briefly indicate the variation of the rate of classification errors of our method with the number of candidate classes. We see that when the number of candidate classes is smaller than that of all the classes, our method almost always obtains a lower rate of classification errors. This clearly shows that to discard the classes that are 'far' from the test sample is really beneficial for the improvement of the accuracy of our method.

## 5. Conclusions

We propose a very promising method to exploit limited training samples for two-step face recognition. The new training samples generated in this paper can well exploit the symmetry structure of the face. The face almost always has a symmetrical structure and the real face image is usually not an axial symmetry image owing to the non-frontal poses, so to use the left (or right) half of the face image to produce the right (or left) half of the face allows us to obtain a possible new sample of the right (or left) half of the face. This is very helpful for overcoming the drawback of limited training samples in the real-world face recognition system. The two-step classification used in the paper can greatly eliminate the side-effect, on the classification of the test sample, of the classes that are 'far' from it (the test sample is usually not truly from these classes). The proposed weighted score level fusion can use a very proper way to integrate the original training samples and 'symmetrical face' training samples for ultimate face recognition by assigning a larger weight to the original training

samples. The experimental results show that the proposed method can outperform a number of state-of-the-art representation based methods.

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